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MECHANICS.

When this issue was made up solutions had been received for numbers 292, 293, 295–6–7–8–9. Please give attention to 272, 277–8–9, 286–7, 290–1, 294, 300.

301. Proposed by CLIFFORD N. MILLS, Brookings, S. D.

A wire is hanging from two points in the same horizontal plane. If the difference between the length of the wire and the actual distance between the supports is very small, show that

$$s = x\left(1 + \frac{x^2}{6c^2}\right),$$

where s is one half the length of the wire, c is the horizontal tension at the lowest point divided by w the load per unit of horizontal distance, and x is the distance of lowest point of the curve to the point of support.

NUMBER THEORY.

When this issue was made up solutions had been received for numbers 212–13, 215–16, 218, 220, 223. Please give attention to 191–2, 196, 198, 201–2, 205, 208–9–10–11, 214, 217, 219, 221–2.

224. Proposed by PATRICK WALSH, New Orleans, Louisiana.

Find the sides, in rational numbers, of a right angled triangle whose area is $5\frac{1}{2}$.

225. Proposed by W. DE W. CAIRNS, Oberlin College.

L'Intermédiaire for June, 1914, contains the following problem:

"If we write the terms of the arithmetic series 1, 5, 9, 13, 17, 21, 25, 29, 33, $\cdots$ as follows:

$$\begin{array}{rrrrrrr}
1 \\
5 & 9 & 13 \\
17 & 21 & 25 & 29 & 33 \\
37 & 41 & 45 & 49 & 53 & 57 & 61 \\
. & . & . & . & . & . & .,
\end{array}$$

it is seen that the sum of the terms of each line is a cube, and that these are the cubes of the successive " odd integers. How is this shown?"

It is here proposed not only to prove this, but to generalize the theorem as suggested, using, however, the simpler (and better known) case which includes all of the successive integers:

$$\begin{array}{rrrr}
1 \\
3 & 5 \\
7 & 9 & 11 \\
13 & 15 & 17 & 19 \\
. & . & . & .
\end{array}$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

412. Proposed by H. L. SLOBIN, University of Minnesota.

Form the algebraic equation whose roots are:

$$a_1 = \cos\frac{\pi}{9}, \qquad a_2 = -\cos\frac{2\pi}{9}, \qquad a_3 = -\cos\frac{4\pi}{9}.$$

Solution by Clifford N. Mills, Brookings, South Dakota.

From the suggestions given by the proposer,

$$\cos r\pi = \frac{(-1)^r + (-1)^{-r}}{2}.$$

Hence,

$$a_1 = \frac{(-1)^{1/9}+(-1)^{-1/9}}{2},\quad a_2 = -\frac{(-1)^{2/9}+(-1)^{-2/9}}{2},\quad a_3 = -\frac{(-1)^{4/9}+(-1)^{-4/9}}{2},$$

and the equation to be constructed is

$$x^3 - (a_1 + a_2 + a_3)x^2 + (a_1a_2 + a_1a_3 + a_2a_3)x - a_1a_2a_3 = 0.$$

We have

$$\Sigma a_1 = \frac{-(-1)^{8/9} - (-1)^{6/9} + (-1)^{5/9} + (-1)^{3/9} - (-1)^{2/9} - 1}{2(-1)^{4/9}},$$

which involves the nine 9th roots of (-1).

From $x^9 + 1 = 0$ we have

$$x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0,$$

which contains all the *complex* 9th roots of (-1).

Hence

$$x^8 + x^6 - x^5 - x^3 + x^2 + 1 = x^7 - x^4 + x,$$

and this is equal to zero since

$$x^7 - x^4 + x = x(x^6 - x^3 + 1),$$

in which the factor $x^6 - x^3 + 1 = 0$.

Therefore

$$\Sigma a_1 = \frac{-(-1)^{1/9}[(-1)^{6/9} - (-1)^{3/9} + 1]}{2(-1)^{4/9}} = 0.$$

Again,

$$a_1a_2 + a_1a_3 + a_2a_3 = -\frac{1}{4}\left[\frac{(-1)^{6/9} + (-1)^{4/9} + (-1)^{2/9} + 1}{(-1)^{3/9}} + \frac{(-1)^{10/9} + (-1)^{8/9} + (-1)^{2/9} + 1}{(-1)^{5/9}} - \frac{(-1)^{12/9} + (-1)^{8/9} + (-1)^{4/9} + 1}{(-1)^{6/9}}\right].$$

Hence

$$\Sigma a_1a_2 = \frac{1}{4}\,\frac{(-1)^{12/9} - (-1)^{11/9} - 2(-1)^{9/9} + (-1)^{8/9} - (-1)^{7/9} - (-1)^{5/9} + (-1)^{4/9} - 2(-1)^{3/9} - (-1)^{1/9} + 1}{(-1)^{6/9}}.$$

Now

$$x^8 - x^7 - x^5 + x^4 - x^3 - x + 1 = -x^6 - x^2.$$

Hence

$$\Sigma a_1a_2 = \frac{1}{4}\,\frac{x^9 - 2x^6 - x^3 + 1}{x^3} = \frac{1}{4}\,\frac{x^9 - 3x^3 + 1}{x^3} = -\frac{3}{4},$$

making use of $x^9 + 1 = 0$ and $x^6 = x^3 - 1$.

Finally,

$$\begin{aligned} a_1a_2a_3 &= \frac{(-1)^{14/9} + (-1)^{13/9} + (-1)^{10/9} + (-1)^{8/9} + (-1)^{6/9} + (-1)^{4/9} + (-1)^{2/9} + 1}{8(-1)^{7/9}} \\ &= \frac{-(-1)^{5/9} - (-1)^{3/9} - (-1)^{1/9} + (-1)^{7/9} + (-1)^{5/9} + (-1)^{3/9} + (-1)^{1/9}}{8(-1)^{7/9}} \\ &= \frac{(-1)^{7/9}}{8(-1)^{7/9}} = \frac{1}{8}, \end{aligned}$$

making use of $x^8 + x^6 + x^4 + x^2 + 1 = x^7 + x^5 + x^3 + x$.

Hence, the required equation is

$$x^3 - \tfrac{3}{4}x - \tfrac{1}{8} = 0.$$

Solved similarly by H. C. Feemster, and also by C. E. Githens by merely substituting the values of a_1, a_2, a_3 and reducing by means of trigonometric relations. Mr. Feemster also sent in such a second solution.

Editorial note. The details of the work on this problem may be simplified by use of Gauss' periods. See Weber's *Algebra*, volume I, chapter XVI.

413. Proposed by C. N. SCHMALL, New York City.

Apply Euler's transformation to show that

$$1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots = \frac{1 + x}{(1 - x)^3}.$$

Solution by W. C. Brenke, University of Nebraska.

This is a special case of the series

$$S(x) = 1 + 2^rx + 3^rx^2 + 4^rx^3 + \cdots;\ r = \text{positive integer.}$$

Such series may be summed by repeated use of Euler's transformation based on the factor $(1 - x)$. The whole operation may be performed at once as follows, the process being valid for $|x| < 1$.

Multiply both sides of the last equation by

$$(1 - x)^{r+1} = 1 - {}_{r+1}C_1x + {}_{r+2}C_2x^2 - \cdots + (-1)^{r+1}x^{r+1}.$$

Arranging the right member in powers of x, we find that the coefficients of x^{r+1}, x^{r+2}, $\cdots$ are all zero by virtue of the formula (special case of (3) p. 183 of Chrystal's Algebra, Vol. 2)

$$n^r - (n - 1)^r{}_nC_1 + (n - 2)^r{}_nC_2 - \cdots + (-1)^{n-1}{}_nC_{n-1} = 0;\ r < n.$$

Hence

$$(1 - x)^{r+1}S(x) = 1 + (2^r - {}_{r+1}C_1)x + (3^r - 2^r{}_{r+1}C_1 + {}_{r+1}C_2)x^2 + \cdots$$
$$+ (r^r - (r - 1)^r{}_{r+1}C_1 + \cdots + (-1)^r{}_{r+1}C_r)x^r.$$

The result may also be written in the form

$$S(x) = \frac{S_r(x) - {}_{r+1}C_1xS_{r-1}(x) + {}_{r+1}C_2x^2S_{r-2}(x) + \cdots + (-1)^r{}_{r+1}C_rx^r}{(1 - x)^{r+1}},$$

where

$$S_n(x) = 1 + 2^rx + 3^rx^2 + \cdots + (n + 1)^rx^n.$$

When $r = 2$ this gives the sum of the proposed series.

Also solved by the Proposer.

414. Proposed by R. D. CARMICHAEL, Indiana University.

Prove by means of an example that one of the series

$$\sum_{k=1}^{\infty} \frac{1}{c_k}, \quad \sum_{k=1}^{\infty} \frac{1}{c_k - 1}, \qquad c_k \neq 0, 1,$$

may be divergent while the other is convergent.